ECONOMETRICS

by

Mareda K. Bell

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INTRODUCTION

Because most economic concepts are quantitative, the growth of mathematical economics was inevitable. Be because economics is not a static system, that is, the value of one or more of the influencing variables can change without an obvious reason, statistical methods are applied to attempt to predict the value of these non-static variables. It is this application of statistical inference to the mathematical formulation of economic theory that is known as econometrics.[1]

Ludwig von Mises contends that the subjectivity and changeability of economics does not allow the mathematician to ply his trade. His main point of contention is that human action cannot be defined as 'behavior constants,' that in fact these 'constants' are manifestly variable; a slight change in human action can effect a proportionately greater change in the end result. Furthermore, the econometrician's hypothetical choice of these 'constants' is arbitrary, made only to cancel the inaccuracies between the fact and the theory.[2] It is my purpose to show that Herr Mises has neglected a very important concept of econometrics, that of probability. The econometrician does realize "that any scientific treatment of problems of value judgements must take into full account the fact that these judgements are subjective and changing."[3] He allows for it and predicts his 'behavioral constant' with accepted methods of statistical inference.

I will outline the structure of econometrics and indicate the methods. I will not attempt to go into a conclusive study of the economic theory, but will elaborate when the explanation will clarify the situation. Furthermore, unless the mathematical notation was unfamiliar to me, I will not go into a discussion of it and will assume the reader is familiar with it. It is not my purpose to study economic theory nor mathematical theory, but to indicate the relationship between the two.

GENERAL DEFINITIONS

An economic theory is called a model, which in turn can be defined as a system of structural equations.[4] In other words, we know of a system of relationships in an economic theory; the statement of this system in mathematical form is a model. The equations of the model can be of two types, definitional or behavioral. Definitional equations are identities or truisms. They contain meither causal relationships nor description of decision processes. For example, "Total expenditures on a commodity equals its price times quantity bought" is a definitional equation. On the other hand, behavioral equations describe the reactions of an economic group to a given set of circumstances. Some examples of behavioral equations are the "supply functions, demand functions and production functions."[5]

The equations are composed of systematic variables, both endogenous and exogenous. Systematic variables "are those to which definite values can be assigned with certainty and which are related to each other in an orderly and systematic way."[6] Endogenous variables are those which are inherent in the model. They are determined by the other variables and determine the other endogenous variables. Exogenous variables are determined by forces outside the model. Although they influence the endogenous variables, they are not influenced by them.[7] In contrast to the systematic variables, we have random variables. A random variable may be defined to be any individual of a population which has exactly the same probability of being selected as any other individual. The selection is independent of the selection of any other individual.[8] The random variable is valuable because it is the statician's tool for explaining any disturbance not explained by changes in the systematic variables.

Schematically, the definitions in the preceding paragraphs are related as follows:

MODEL
A Complete System of:

- STRUCTURAL EQUATIONS
  - Definitional Equations
  - Behavioral Equations consist of variables which are
    - Systematic
      - Exogenous
      - Endogenous
    - Random [9]

The model is considered complete when the number of equations is at least equal to the number of endogenous variables, given values for the exogenous variables.

There are four basic model types, (1) static certainty, (2) dynamic certainty, (3) static uncertainty, and (4) dynamic uncertainty. If the econometrician wishes to abstract his model from time, consider any changes to be instantaneous, and the effect of these changes also to be instantaneous, he will establish a 'static model.' If, on the other hand, he considers time as a component of his model and takes notice of the effect of time on the endogenous variables, he creates a 'dynamic model.' Certainty and uncertainty pertain to the 'degree of knowledge'. In the first case, the persons involved are considered to have perfect knowledge of the subject and its effects on the economy. This is to add a degree of simplicity. In the second case, these persons are assumed to have an imperfect knowledge of these conditions but have an idea of the prevailing or expected conditions. Decisions will then be made on the "basis of information which is probable rather than certain."[10] Static certainty is the simplest of these with which to work; while dynamic uncertainty is the most realistic. For our puposese, static uncertainty will be the model most often used.

THE DISTURBANCE VARIABLE

Because the main argument against econometrics is directed against the random variable which serves to allow for all varying forces which are not considered in the model and for the unpreddictable but probable actions of the human element, further study of this concept is necessary.

> A main task of econometric theory, indeed, is to provide a bridge between the exact relationships of economic theory and the disturbed relationships of economic reality.[11]

For example, let $c = \alpha + \beta y$, where c is the consumption expenditure and y is disposable income. α and β are parameters. We have available 3,000 observations on the above variables. The 3,000 paired observatiosn c_t, y_t (t=1,2... 3,000) will not necessarily satisfy $c_t = \alpha + \beta y_t$.[12]

> ...the theoretical relationship does not purport to describe the behavior of every individual household but rather to describe average behavior in some sense.[13]

If this is recognized, we can write $c = \alpha + \beta y + \varepsilon$ where ε is a 'disturbance term' which has been discovered by accepted statistical procedures.[14]

Introduction of the disturbance variable is not sufficient "to make an economic theory operational." If we write $c = \alpha + \beta y + \varepsilon$ and do not make any statement concerning ε other than it is an unobserved variable, then any pair of values for α and β will give us a relationship to fit the data. The statement $\varepsilon_t = c_t - (\alpha + \beta y_t)$ is "tautological and empirical." We must, therefore, make some statement concerning the stochastic qualities of the disturbance term; for example, "each ε_t is a random drawing from a probability distribution with mean zero and finite variance." In some cases, assumptions concerning the distribution are needed.[15]

This disturbance term may be interpreted in one of three ways. First, if we say that consumption is determined exactly as a function of income and other factors such as the number of persons in the household, ages of the members, etc., then differences at a given income level are attributable to the values of these other variables. The disturbance variable is introduced to represent the net effect of these omitted variables.[16] A second interpretation is that the disturbance variable can represent the "inherent indeterminancy of human behavior."[17] Third, although there may be an exact relationship among the variables, the observations may be inaccurate. In this case, ε is a reflection of these inaccuracies.[18]

ECONOMIC MODELS AND MATHEMATICS

Before we are able to apply statistical procedure to economic situations, we must be able to state the theory in mathematical terms. The important aspect of mathematical economics is that this symbolic representation "does not improve the accuracy of analysis, but it does increase the precision."[19]

As I said before, an econometric model is the mathematical statement of an economic theory. The purpose of reducing theory to abstract variables is to enable us to more precisely solve simultaneously the equations which are inherent to the statement of this economic theory. If the equations can be solved we will have an acceptable model. For instance, by relating the demand and supply equations in a model, we may make predictions concerning equilibrium values of the price and quantity of a given product by determining the values of the variables representing these factors.[20]

An example of a static certainty model may be outlined in formal notation as follows:

$$\begin{aligned} x_d &= f(p_x, p_o, y) && (1)\\ x_s &= g(p_x, w) && (2)\\ p_o &= \overline{p}_o && (3)\\ y &= \overline{y} && (4)\\ w &= \overline{w} && (5)\\ x_d &= x_s \text{ [21]} && (6) \end{aligned}$$

Let x_d represent the quantity demanded in millions of units per year, and x_s be the quantity supplied in millions of units per year. p_x denotes the price of product X in dollars per unit, and p_o indicates the price of a substitute product in dollars per unit. Finally, y indicates the consumer income in billions of dollars, and w is the wage rate of labor (in billions of dollars) used to produce the commodity X. p_o, y, and w are exogenous variables and constant (indicated by the bar notation, $\overline{p_o}$). Equations (1) through (5) are behavioral, and equation (6) is definitional.[22] Equation (6) indicates that we want an equilibrium situation. Even in this formal notation, we can see that there is a solution to this economic model.

Now let us assume that we know the equations for these functions.

$$\begin{aligned} x_d &= 310 - 20p_x - 2p_o + .2y && (1)\\ x_s &= 25 + .10p_x - .1w && (2)\\ p_o &= 4 && (3)\\ y &= 200 && (4)\\ w &= 10 && (5)\\ x_d &= x_s && (6) \text{ [23]} \end{aligned}$$

Solving, we can see

$$x_d = 310 - 20p_x - 2(4) + .2(200) \qquad (1')$$
$$x_s = 25 + 10p_x - .1(10) \qquad (2')$$
$$x_d = x_s \qquad (6)$$

or

$$310 - 20p_x + 32 = 25 + 10p_x - 1$$
$$342 - 20p_x = 24 + 10p_x$$
$$30p_x = 318$$
$$p_x = 10.60$$

From this, $x_d = x_s = 24 + 10(10.60) = 130$

The equilibrium price is $10.60 per unit and the equilibrium quantity traded (demanded and supplied) is 130 million units per year.

This is one example of a supply and demand model. It is important to remember that econometric models are only tools; they can be modified or revised to serve the purpose of the economist.[24] Another economist may, depending on his information, alter the equation to reflect his data. If he can support these variations, the model can be accepted as valid.

Because static models 'happen' at a given point in time, a method of comparative statics has been developed to indicate a change in a given parameter. Given a consumption function and an investment function as outlined in the following model

$$C = 15 + .6Y + .1A \qquad (1)$$
$$I = 80 - 1000r \qquad (2)$$
$$C = Y + I \qquad (3)$$
$$A = 100 \qquad (4)$$
$$r = .04 \qquad (5)$$ [25]

it is not difficult to ascertain that C = 122.5, Y = 162.5, and I = 40. In this model, C is consumption; Y is income; A is total assets of households; I is investment; and r is the rate of interest. A, C, I and Y are all in billions of dollars. If the rate of interest changes to .06, and the total assets of households increases to 150, then C will equal 105; Y will equal 125.0; and I will equal 20.

A comparison of these values will yield the investment multiplier:

$$k = \frac{\Delta Y}{\Delta I}.$$ [26]

This ratio indicates "...how great an increase in income results from each increase in investment."[27] In the above case, k = 1.875; or one invested dollar will yield through reinvestment $1.88 in income.

STATISTICAL INFERENCE

In the previous section, we arbitrarily assigned parametric values to the variables in our model. Although this was acceptable for our purposes at that time, we will now attempt to determine the values of these parameters by statistical inference. Included in this study will be the disturbance variable ε which we discussed previously because ε itself is a parameter of the mathematical sentence.

Although econometric theory draws heavily on mathematical statistics, there are two main distinctions: (1) Observations of economic phenomena are not obtained by controlled experiment; special methods of analysis for this data must be devised. (2) Special methods are developed to take advantage of the "rich body of economic behavior" which may substitute for experimental control.[28] Because we know the relationship among the variables we can relate our seemingly scattered data to a concept rather than attempting to find a relationship for our random data.[29]

Before I go into the application, I will discuss here the method of least squares which will be important in the discovery of the parameters. If our observed data consist of n pairs (X_1, Y_1), $(X_2, Y_2)\ldots(X_n, Y_n)$, we will wish to find a linear relationship between these values (if one exists) such that

$$Y' = \alpha + \beta X$$

where Y' is the expected value of Y. The deviation from the actual value of Y may be defined

$$Y - Y' = Y - \alpha - \beta X.$$ [30]

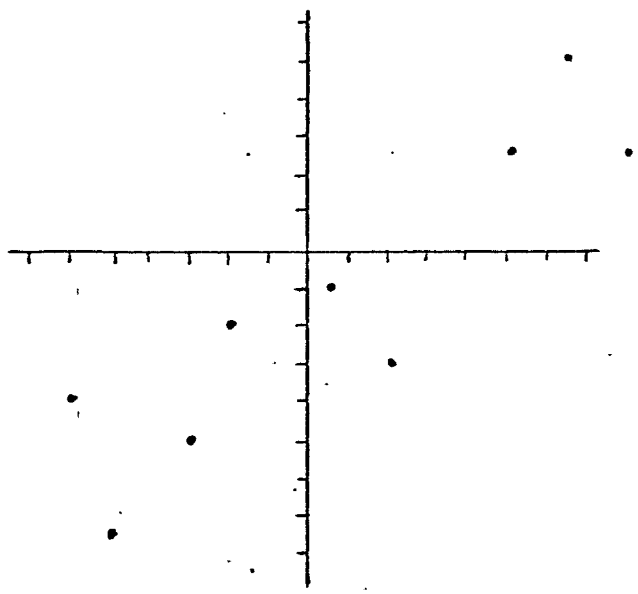

FIGURE I [31]

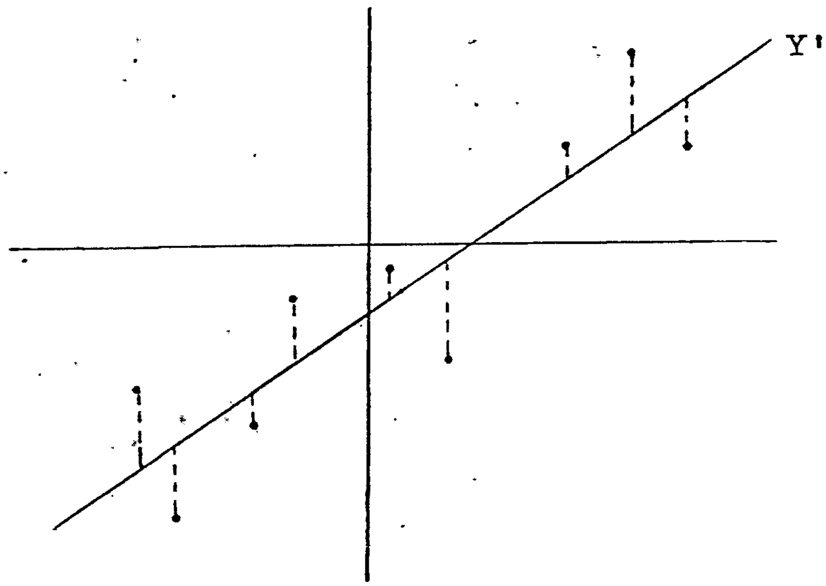

FIGURE II [32]

If Figure I is the graphic representation of our observed data, we can construct a line which approximates this data (Figure II). The deviations will be the vertical distances from the points to the line.

"The principle of least squares says that a desirable approach (to the construction of this line) is to minimize the sum of the squares of the deviations."[33] To do this, we sum the deviations and set the sum equal to zero.

$$\Sigma(Y - \alpha - \beta X) = \Sigma Y - n\alpha - \beta\Sigma X = 0$$

If we divide by n,

$$\overline{Y} - \alpha - \beta\overline{X} = 0$$

where n is the number of observations, and $\overline{X}$ and $\overline{Y}$ are the mean values of the samples. Let S be the sum of the squares of the deviations.

$$S = \Sigma(Y - \alpha - \beta X)^2$$

$$S = \Sigma(Y^2 + \alpha^2 + \beta^2X^2 - 2\alpha Y - 2\beta XY + 2\alpha\beta X)$$

$$S = \Sigma Y^2 + n\alpha^2 + \beta^2\Sigma X^2 - 2\alpha\Sigma Y - 2\beta\Sigma XY + 2\alpha\beta\Sigma X.$$

We may minimize this by taking the partial derivative with respect to α and β.[34]

$$\frac{\partial S}{\partial \alpha} = 2n\alpha - 2\Sigma Y + 2\beta\Sigma X = 0$$

or

$$n\alpha + \beta\Sigma X - \Sigma Y = 0$$

$$\frac{\partial S}{\partial \beta} = 2\beta\Sigma X^2 - 2\Sigma XY + 2\alpha\Sigma X = 0$$

or

$$\alpha\Sigma X + \beta\Sigma X^2 - \Sigma XY = 0$$

The resulting equations are called the normal equations, and the first requires that the line Y' pass through $(\overline{X}, \overline{Y})$. The resulting line is known as the 'least squares line' or the 'regression line.'[35] If we solve for α and β.

$$\alpha = \frac{\Sigma Y - \beta\Sigma X}{n}$$

$$\left[\frac{\Sigma Y - \beta\Sigma X}{n}\right]\Sigma X + \beta\Sigma X^2 - \Sigma XY = 0$$

$$\Sigma X\Sigma Y - \beta(\Sigma X)^2 + n\beta\Sigma X^2 - n\Sigma XY = 0$$

$$n\beta\Sigma X^2 - \beta(\Sigma X)^2 = n\Sigma XY - \Sigma X\Sigma Y$$

$$\beta = \frac{n\Sigma XY - \Sigma X\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$n\alpha + \left[\frac{n\Sigma XY - \Sigma X\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}\right]\Sigma X - \Sigma Y = 0$$

$$n\alpha = \Sigma Y - \frac{n\Sigma X\Sigma XY - (\Sigma X)^2\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$\alpha = \frac{1}{n}\left[\frac{n\Sigma X^2\Sigma Y - (\Sigma X)^2\Sigma Y - n\Sigma X\Sigma XY + (\Sigma X)^2\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}\right]$$

$$\alpha = \frac{\Sigma X^2\Sigma Y - \Sigma X\Sigma XY}{n\Sigma X^2 - (\Sigma X)^2}$$ [36]

THE COEFFICIENT OF CORRELATION AND A TEST FOR SIGNIFICANCE

Before we can accept these values of α and β, we must test them to determine whether or not they adequately reflect the distribution of the sample.

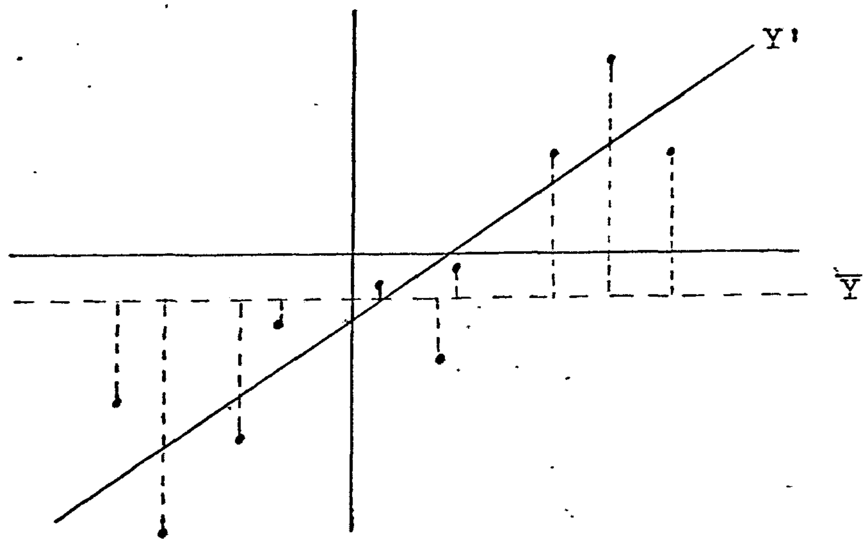

FIGURE III[37]

Figure III (which is the same graph that appears on page 10) reflects the deviations of y_i from the mean value $\overline{Y}$. The test which is outlined below is simply a comparison of the sum of the squares of deviations from the line of regression $\sum(Y - Y')^2$ to the sum of the squares of the deviations from the mean $\sum(Y - \overline{Y})^2$. We will then establish a null hypothesis and test the sampling distribution of r by establishing confidence limits on a normal distribution.[38]

The coefficient of correlation is "the most widely used measure of the strength of the linear relationship between two variables."[39] Let us square the identity:

$$Y - \overline{Y} = (Y - Y') + (Y' - \overline{Y}).$$

$$(Y - \overline{Y})^2 = (Y - Y')^2 + 2(Y - Y')(Y - \overline{Y}) + (Y' - \overline{Y})^2$$

If we sum over all the values in the sample, our equation becomes

$$\Sigma(Y - \overline{Y})^2 = \Sigma(Y - Y')^2 + \Sigma(Y' - \overline{Y})^2. \quad 40$$

If we divide the first normal equation, $\alpha n + \beta \Sigma X - \Sigma Y = 0$ by n, we obtain

$$\overline{Y} = \alpha + \beta \overline{X}. \quad 41$$

Also we have defined $Y' = \alpha + \beta X$; therefore, it follows that

$$(Y' - \overline{Y}) = \alpha + \beta X - (\alpha + \beta \overline{X}) = \beta (X - \overline{X}).$$

Therefore, $$\Sigma(Y' - Y)^2 = \beta^2 \Sigma (X - X)^2. \quad 42$$

We define the coefficient of correlation to be

$$r = \sqrt{\frac{\Sigma (Y' - Y)^2}{\Sigma (Y - Y)^2}}, \quad 43$$

but we have already shown that $\Sigma(Y' - \overline{Y})^2 = \beta^2 \Sigma(X - \overline{X})^2$.

$$r = \pm \beta \sqrt{\frac{\Sigma (X - X)^2}{\Sigma (Y - Y)^2}}$$

As r approaches 0, the linear relationship between x and y becomes weaker; and as r approaches ±1, the relationship becomes stronger. Further, as r approaches ±1, the greater percent of change in y is attributable to change in x. That is, the force of the disturbance variable becomes weaker.[44]

In general, in economic research, the samples will be large; therefore, we can establish a null hypothesis that there is no relationship between X and Y. Also the sampling distribution of r can be approximated closely with a normal curve with mean zero and standard deviation $1/\sqrt{n - 1}$ because the observations of X and Y are taken from a normal population.[45]

Let λ equal the level of significance (usually 0.10, 0.05, or 0.01 [46]) which has been predetermined. We normalize the value of r according to the formula:

$$z = \frac{r - \mu}{\sigma}$$

which in our case will be

$$z = r\sqrt{n - 1}\ .$$ [47]

We then set up our confidence limits by determining that part of the normal distribution which will allow us to accept the null hypothesis. In the problem following, we will set $\lambda = .05$. Because the area under the normal curve is 1, the area of the region of acceptance will be .95 (or 95% of the area under the curve). We halve this in order to use the 'Table of Areas under the Unit Normal Curve.'[48] In this case the region of acceptance is $-1.96 \leqslant z \leqslant 1.96$.

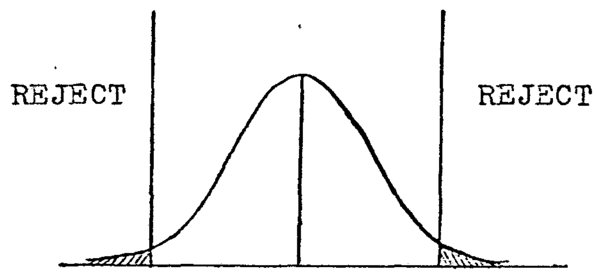

FIGURE IV[49]

The rejection of the null hypothesis as outlined above is a confirmation of the values of α and β because the coefficient of correlation is derived by using the sum of the squares of the deviations from the regression line.

OBJECTIONS TO THE COEFFICIENT OF CORRELATION

The main objection to the use of the coefficient of correlation is that, although it does measure association between the variables, it does not prove causation. That is, it does not prove that the dependent variable in actuality is dependent on the independent variable. For example, if we have two sample distributions X and Y such that X is "the annual per capita consumption of chewing tobacco in the United States from 1948 through 1955," and Y is the "number of auto thefts reported in a sample of urban areas in the same years,"[50] we may be able to prove a linear relationship between the two distributions. However, we cannot state that Y is dependent on X; the correlation may be due to the "dependence of both variables on social patterns changing with time."[51] This must be verified by further analysis.

The objection can be resolved by the knowledge of economic theory. The econometrician knows beforehand that there is a relationship between the variables; his purpose is to show the exact magnitude of this relationship.

NUMERICAL ILLUSTRATION

Problem:[52]

The following sample of observations on the price and quantity exchanges of a commodity X is given. From this information, (a) construct a scatter diagram and establish the form of the population demand equation for X, (b) estimate the parameters of the demand equation and test them.

COMMODITY X			
PRICE	QUANTITY	PRICE	QUANTITY
10	110	35	55
8	98	25	60
20	75	28	55
16	100	30	65
24	80	35	80
30	58	40	30
36	54	45	40
40	40	40	40

Solution:

The scatter diagram is below and seems to indicate that the form of the equation should be

$$q = \alpha + \beta p_{.53}$$

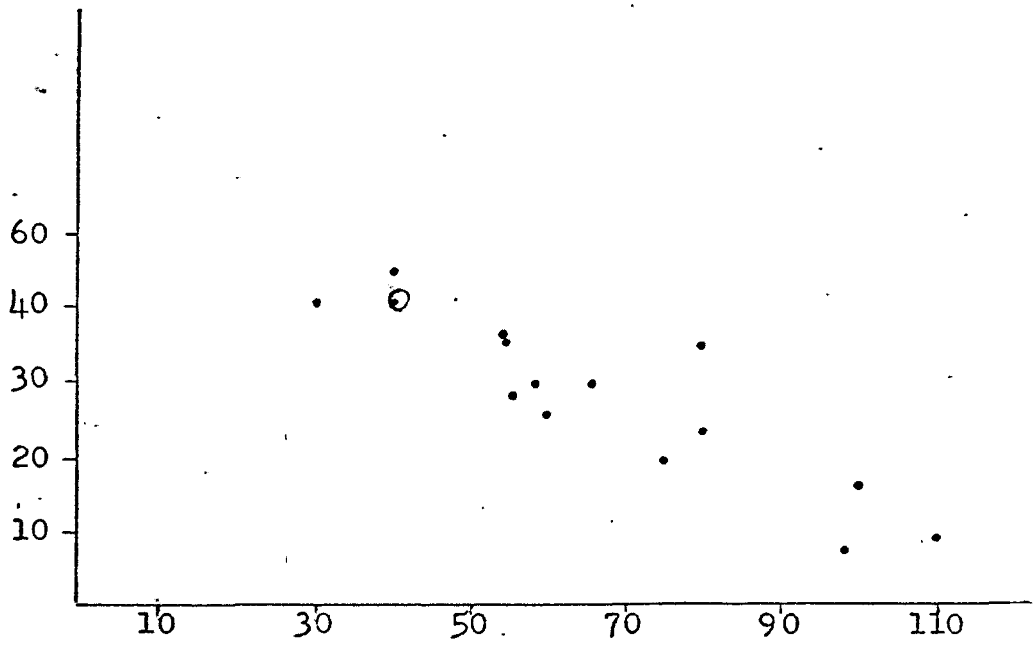

FIGURE V

⊙ Indicates two points.

q	p	p^2	pq
110	10	100	1100
98	8	64	784
75	20	400	1500
100	16	256	1600
80	24	576	1920
58	30	900	1740
54	36	1296	1944
40	40	1600	1600
55	35	1225	1925
60	25	625	1500
55	28	784	1540
65	30	900	1950
80	35	1225	2800
30	40	1600	1200
40	45	2025	1800
40	40	1600	1600
$\Sigma q = 1040$	$\Sigma p = 462$	$\Sigma p^2 = 15176$	$\Sigma pq = 26503$
$\bar{q} = 65$	$\bar{p} = 28.875$	$n = 16$	

TABLE I

$$\alpha = \frac{\Sigma p^2 \Sigma q - \Sigma p \Sigma pq}{n \Sigma p^2 - (\Sigma p)^2} = \frac{(15{,}176)(1040) - (462)(26{,}503)}{(15{,}176)(16) - (462)^2}$$

$$\alpha = \frac{15{,}783{,}040 - 12{,}244{,}386}{242{,}816 - 213{,}444} = \frac{3{,}538{,}654}{29{,}372}$$

$$\alpha = 120.48$$

$$\beta = \frac{n \Sigma pq - \Sigma p \Sigma q}{n \Sigma p^2 - (\Sigma p)^2} = \frac{16(26{,}503) - (462)(1040)}{16(15{,}176) - (462)^2}$$

$$\beta = \frac{424{,}048 - 480{,}480}{242{,}816 - 213{,}444} = \frac{-56{,}432}{29{,}372}$$

$$\beta = (-1.92)$$

The relationship can now be stated:

$$q = 120.48 - 1.92p$$

Test for Significance:

$$r = \beta\sqrt{\frac{\Sigma(p - \bar{p})^2}{\Sigma(q - \bar{q})^2}}$$

p - p	(p - p)^2	q - q	(q - q)^2
-18.9	357.21	45	2025
-20.9	436.81	33	1089
- 8.9	79.21	10	100
-12.9	166.41	35	1225
- 4.9	24.01	15	225
1.1	1.21	- 7	49
7.1	50.41	-11	121
11.1	123.21	-25	625
6.1	37.21	-10	100
- 3.9	15.21	- 5	25
- .9	.81	-10	100
1.1	1.21	0	0
6.1	37.21	15	225
11.1	123.21	-35	1225
16.1	259.21	-25	625
11.1	123.21	-25	625
	(p - p)^2 = 1835.76		(q - q)^2 = 8384

TABLE II

$$r = -1.92\sqrt{\frac{1835.76}{8384}} = -1.92\sqrt{.218}$$

$$r = (-1.92)(.467) = -.896$$

To test whether or not r is significant, let

$$z = -(.896)\sqrt{16 - 1}$$

$$z = -(.896)\sqrt{15}$$

$$z = -(.896)(3.87) = -3.47$$

Because the null hypothesis states that there is no correlation between q and p if $-1.96 \leq z \leq 1.96$, our value of z allows us to reject this hypothesis for a 95% significance level.

Furthermore, according to our statistic 89% of the variation in q is due to variation in p.

CONCLUSION

I have restricted this discussion to linear regression for two reasons. In the first place, this paper was intended to serve is an introduction to econometrics and to have expanded the discussion to curvilinear regression would have caused the basics to become lost in the details. Second, the economic theory would have become almost as involved as the mathematics.

The economist dislikes applying mathematics to his field because it restricts his freedom, or so he feels. The mathematician doesn't want to attempt applying his field to economics because the latter field is so uncertain and he can not obtain the precision and accuracy which is inherent to his field. However, both must admit that this intermingling is not only necessary, it is inevitable. Eventually, the economist must admit that his relationships have reached a degree of difficulty which makes it practically impossible for him to handle. Mathematics is then his tool. On the other hand, the mathematician must allow that the field of mathematical statistics makes allowances for the inaccuricies of economic data and his dislike of these inaccuracies can be somewhat alleviated.

While maintaining simplicity, I attempted to show that the two fields could be related. It is known that the price of a commodity has a negative effect on the quantity demanded. Therefore, we have another indication that our statement of the relationship between p and q is valid. However, the reader should realize that this relationship is used here only for purposes of illustration. In the first place, the factors affecting the demand function usually cause it to be curvilinear (This is the relationship usually accepted by economists.), Second, our parameters were not derived from a large enough sample to be considered valid. Much research is conducted before the econometrician even begins his calculations.

I feel that, if properly used, the field of econometrics is pertinent to the current economic scene. However, the results must be recognized as approximations, and adequate consideration of the variations between theory and reality must be made.

FOOTNOTES

[1] Michael J. Brennan, Preface to Econometrics: An Introduction to Quantitative Methods in Economics (Cincinnati: South-Western Publishing Company, 1965), p.10.

[2] Ludwig von Mises, Theory and History, An Interpretation of Social and Economic Evolution (New Haven: Yale University Press, 1957), pp. 10-11.

[3] Ibid., p. 35

[4] Brennan, p. 10.

[5] Ibid., p. 10.

[6] Ibid., p. 11.

[7] Ibid., pp. 10-11.

[8] Ibid., p. 12.

[9] Ibid., p. 13.

[10] Ibid., pp. 14-15.

[11] Arthur S. Goldberger, Econometric Theory (New York: John Wiley & Sons, Inc., 1964), p. 2.

[12] Ibid., p. 2.

[13] Ibid., p. 2.

[14] Ibid., pp. 2-3.

[15] Ibid., pp. 3-4.

[16] Ibid., pp. 2-3.

[17] Ibid., p.3.

[18] Ibid., p. 3.

[19] James L. Riggs, Economic Decision Models (New York: McGraw-Hill Book Company, 1968), p. 17.

[20] Brennan, p. 207.

[21] Ibid., p. 216 and pp 224-5.

[22] Ibid., p. 216-7.

[23] Ibid., pp. 224-5.

[24] Ibid., p 224.

25 *Ibid.*, p. 225.

26 *Ibid.*, p. 223.

27 Paul A. Samuelson, *Economics, An Introductory Analysis* (New York: McGraw-Hill Book Company, Inc., 1951), p. 280.

28 Goldberger, p. 1.

29 'Scattered' was used in the first case to differentiate between the uncontrolled experimental data and the 'random sample' found by a controlled experiment.

30 Howard W. Alexander, *Elements of Mathematical Statistics* (New York: John Wiley & Sons, Inc. 1961), p. 288.

31 Brennan, p. 310.

32 *Ibid.*, p. 310

33 Alexander, p. 288.

34 *Ibid.*, pp. 288-9.

35 *Ibid.*, pp. 288-9.

36 *Ibid.*, p. 289.

37 Brennan, p. 310

38 John E. Freund, *Modern Elementary Statistics* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960), pp. 326-338.

39 *Ibid.*, p. 328.

40 The term $\Sigma(Y - Y')(Y' - \overline{Y})$ can be shown to equal 0 by use of the normal equations.

41 $\overline{X} = (1/n)\Sigma X$ and $Y = (1/n)\Sigma Y$.

42 Alexander, pp. 292-3.

43 *Ibid.*, p. 295.

44 Freund, p. 334.

45 *Ibid.*, p. 337.

46 Goldberger, p. 139.

47 Freund, p. 337.

48 Alexander, p. 343.

[49] Freund, p. 337.

[50] *Ibid.*, p. 329.

[51] *Ibid.*, p. 336.

[52] as stated in Brennan, pp. 333-4.

[53] Contrary to mathematical convention which places the dependent variable on the vertical axis, economic graphs place the quantity demanded on the horizontal axis and the price on the vertical. Brennan, p. 55.

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